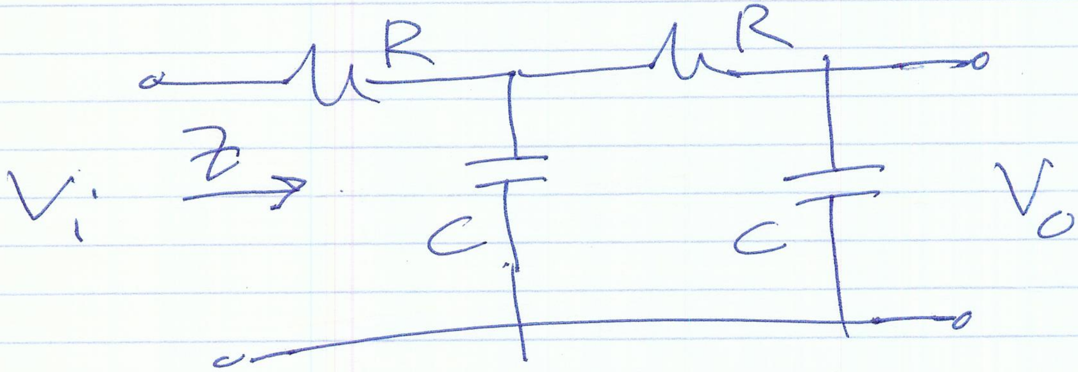
Lab 2: Notes on the cascaded circuit

Nicole Hamilton

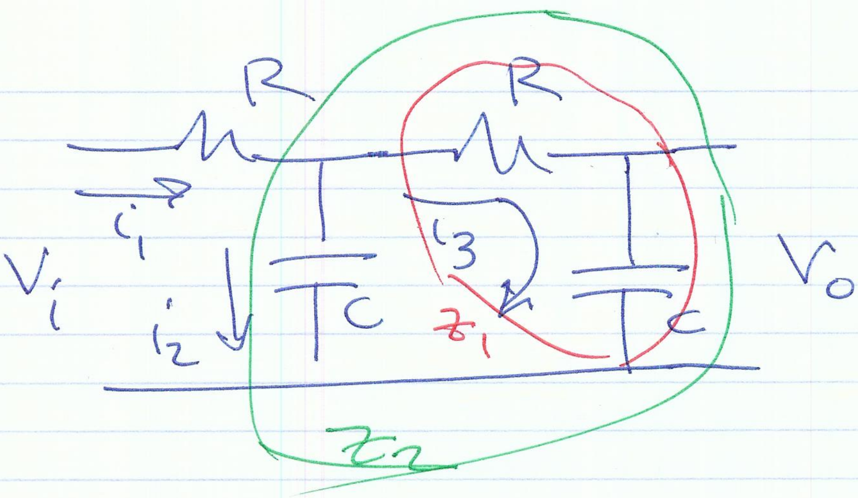
The cascaded circuit is called a second-order circuit because it has 2 energy storage devices, i.e., 2 capacitors.



We expect the characteristic equation to be of the form

$$V_o = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_0$$

To solve this, I'll write a polynomial equation for Z looking in, then find the roots where $Z \to 0$ to determine the natural response, giving me s_1 and s_2. I'll use initial and final conditions to determine the A_i values.



$$z_1 = R + \frac{1}{sC}$$

$$Y_1 = \frac{1}{R + \frac{1}{sC}} = \frac{sC}{sRC+1}$$

$$Y_2 = sC + Y_1 = sC + \frac{sC}{sRC+1}$$

$$= \frac{s^2RC^2 + 2sC}{sRC+1}$$

$$z_2 = \frac{sRC+1}{s^2RC^2+2sC}$$

$$z = R + z_2 = R + \frac{sRC+1}{s^2RC^2+2sC}$$

$$z = \frac{s^2R^2C^2 + 3sRC + 1}{sRC^2+2sC}$$

The natural response for this circuit will be given by the roots of the equation for Z, i.e., where $Z \to 0$.

$$Z = \frac{s^2R^2C^2 + 3sRC + 1}{sRC^2 + 2sC} = 0$$

$$s^2R^2C^2 + 3sRC + 1 = 0$$

$$s = \frac{-3RC \pm \sqrt{9R^2C^2 - 4R^2C^2}}{2R^2C^2}$$

For $R = 10K$ and $C = 0.01\,\mu F$

$s_1 = -3{,}819.66$ radians/sec

$s_2 = -26{,}180.34$ radians/sec

$$V_o = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_0$$

We now know s_1 and s_2.

A_0 is given by inspection that as $t \to \infty$, $V_o \to V_i$

$\therefore A_0 = V_i$

To find A_1 and A_2 we need to consider the initial conditions for V_o and dV_o/dt at $t = 0+$.

$V_o\big|_{0+} = 0$ because both capacitors are uncharged

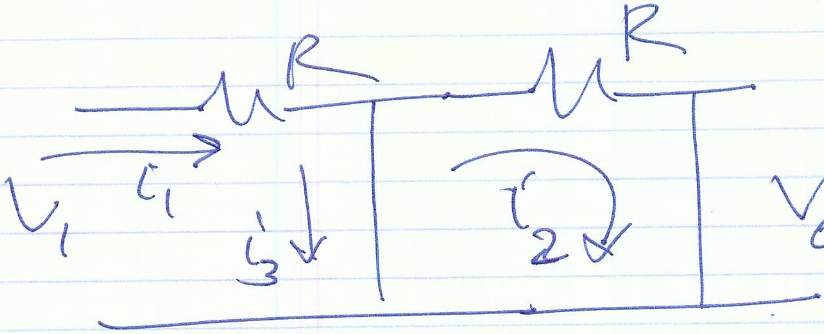
$\therefore A_1 + A_2 + V_i = 0$

$$A_2 = -(A_1 + V_i)$$

$$\frac{dV_o}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

$$\frac{dV_o}{dt}\bigg|_{0+} \text{ must} = i_2 C\bigg|_{0+}$$

At $t=0+$, i_2 must be zero because with both capacitors discharged, they look like shorts:



$i_3 = i_1$

$\therefore i_2 = 0$

$$\left.\frac{dV_o}{dt}\right|_{0+} = A_1 s_1 + A_2 s_2 = i_2 C \Big|_{0+} = 0$$

Substituting $A_2 = -(A_1 + V_i)$

$$A_1 s_1 - (A_1 + V_i) s_2 = 0$$

$$A_1 s_1 - A_1 s_2 - V_i s_2 = 0$$

$$A_1 (s_1 - s_2) = V_i s_2$$

$$A_1 = V_i \frac{s_2}{s_1 - s_2}$$

$$A_2 = -\left(V_1 \frac{s_2}{s_1 - s_2} + V_i\right)$$

$$= -V_i \left(\frac{s_2}{s_1 - s_2} + 1\right)$$

We now have the complete characteristic equation

$$V_0 = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_0$$

$$A_0 = V_i$$

$$s_1 = -3819.66$$

$$s_2 = -26{,}180.34$$

$$A_1 = V_i \left(\frac{s_2}{s_1 - s_2} \right) = -1.171\, V_i$$

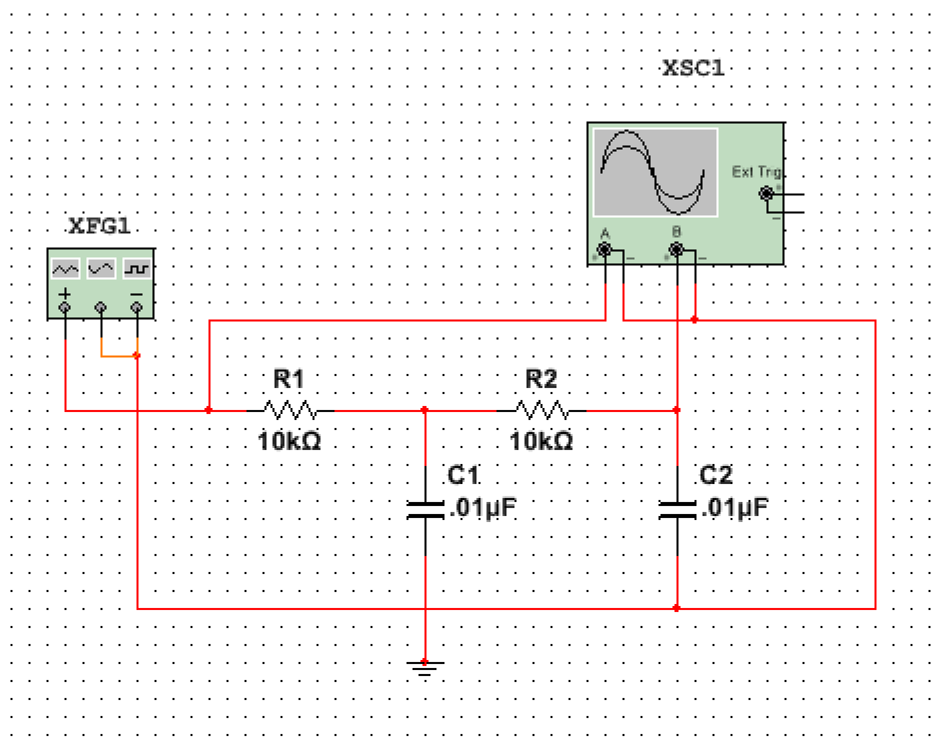
$$A_2 = -V_i \left(\frac{s_2}{s_1 - s_2} + 1 \right) = 0.171\, V_i$$

$$\frac{V_0}{V_i} = -1.171 e^{-3819.66t} + 0.171 e^{-26180.34t} + 1$$

To find t_{PLH}, I solved numerically for $\frac{V_0}{V_i} = 0.5$.

$$t_{PLH} = 222.49\, \mu s$$

Here was my simulation.



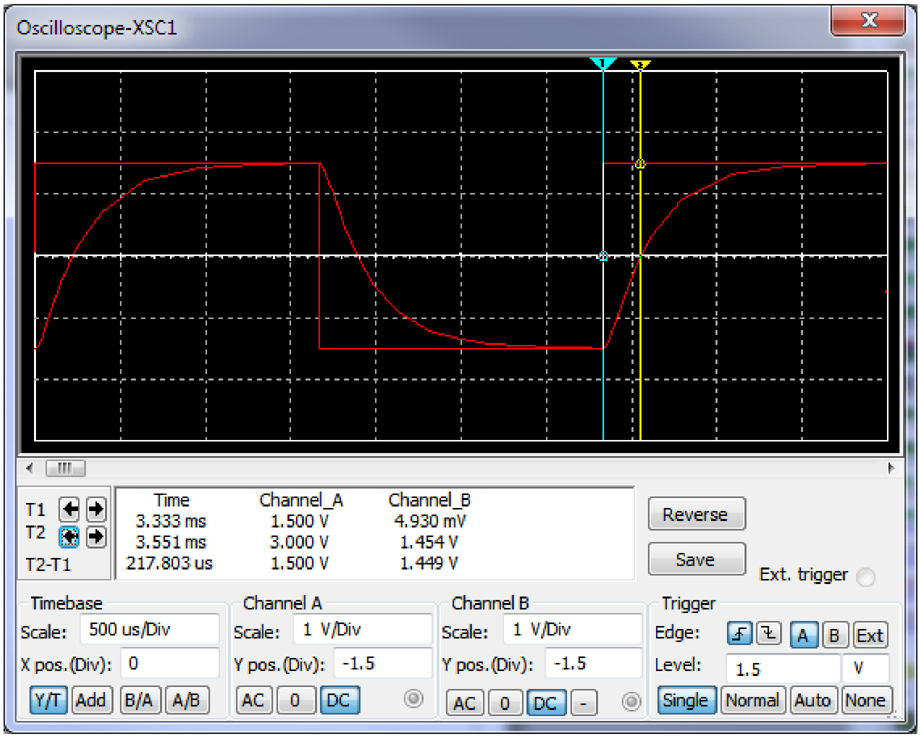
My calculated value for t_{PLH} matches exactly with the granularity of the simulation, seen on the next page.

	Calculated	Simulated
t_{PLH}	222.49 µs	Between 217.803 and 227.273 µs

Here is a comparison of the calculated delay times for the simple (from the prelab) and cascaded circuits.

	Simple	Cascaded	Ratio
t_{PLH}	69.31 µs	222.49 µs	3.21

t_{PLH} > 217.803 µs



t_{PLH} < 227.273 µs